Puntos máximos y mínimos locales

Comprobación de máximos y mínimos

Mediante el cambio de signo de la derivada primera y mediante el signo de la derivada segunda

Uso de los términos “cóncava hacia arriba” para $f''(x) > 0$ y “cóncava hacia abajo” para $f''(x) < 0$

Aplicación: beneficios, áreas, volúmenes

Puntos de inflexión con pendiente nula y no nula
En un punto de inflexión, $f''(x) = 0$ y cambia el signo (cambia la concavidad).
$f''(x) = 0$ no es una condición suficiente para que exista un punto de inflexión: por ejemplo, $y = x^4$ en $(0,0)$.
Comportamiento de los gráficos de las funciones, incluida la relación entre los gráficos de f, f' y f''
Optimización

Example 2 — **Self Tutor**

A particle moves in a straight line with position relative to O given by $s(t) = t^3 - 3t + 1$ cm, where t is the time in seconds, $t \geqslant 0$.

- **a** Find expressions for the particle's velocity and acceleration, and draw sign diagrams for each of them.
- **b** Find the initial conditions and hence describe the motion at this instant.
- **c** Describe the motion of the particle at $t = 2$ seconds.
- **d** Find the position of the particle when changes in direction occur.
- **e** Draw a motion diagram for the particle.
- **f** For what time interval is the particle's speed increasing?
- **g** What is the total distance travelled in the time from $t = 0$ to $t = 2$ seconds?

8 A particle P moves along the x-axis with position given by $x(t) = 1 - 2\cos t$ cm where t is the time in seconds.

- **a** State the initial position, velocity, and acceleration of P.
- **b** Describe the motion when $t = \frac{\pi}{4}$ seconds.
- **c** Find the times when the particle reverses direction on $0 < t < 2\pi$, and find the position of the particle at these instants.
- **d** When is the particle's speed increasing on $0 \leqslant t \leqslant 2\pi$?

TAREA

6 The position of a particle moving along the x-axis is given by $x(t) = t^3 - 9t^2 + 24t$ metres where t is in seconds, $t \geqslant 0$.

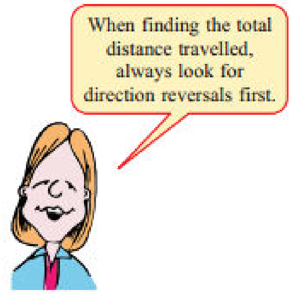
a Draw sign diagrams for the particle's velocity and acceleration functions.

b Find the position of the particle at the times when it reverses direction, and hence draw a motion diagram for the particle.

c At what times is the particle's:

  i speed decreasing **ii** velocity decreasing?

d Find the total distance travelled by the particle in the first 5 seconds of motion.



7 A particle P moves in a straight line with displacement function $s(t) = 100t + 200e^{-\frac{t}{5}}$ cm, where t is the time in seconds, $t \geqslant 0$.

a Find the velocity and acceleration functions.

b Find the initial position, velocity, and acceleration of P.

c Discuss the velocity of P as $t \to \infty$.

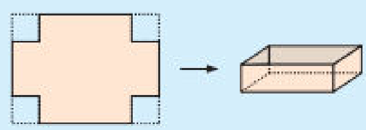
d Sketch the graph of the velocity function.

e Find when the velocity of P is 80 cm per second.

Example 6 Self Tutor

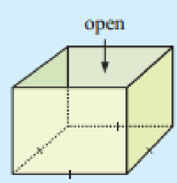
A rectangular cake dish is made by cutting out squares from the corners of a 25 cm by 40 cm rectangle of tin-plate, and then folding the metal to form the container.

What size squares must be cut out to produce the cake dish of maximum volume?



Example 7 Self Tutor

A 4 litre container must have a square base, vertical sides, and an open top. Find the most economical shape which minimises the surface area of material needed.

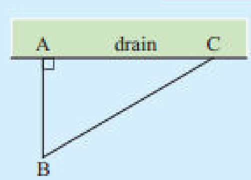


Example 8 Self Tutor

An animal enclosure is a right angled triangle with one side being a drain. The farmer has 300 m of fencing available for the other two sides, [AB] and [BC].

a If $AB = x$ m, show that $AC = \sqrt{90\,000 - 600x}$.

b Find the maximum possible area of the triangular enclosure.



TAREA

2 A duck farmer wishes to build a rectangular enclosure of area 100 m^2. The farmer must purchase wire netting for three of the sides as the fourth side is an existing fence. Naturally, the farmer wishes to minimise the length (and therefore cost) of fencing required to complete the job.

a If the shorter sides have length x m, show that the required length of wire netting to be purchased is $L = 2x + \frac{100}{x}$.

b Use *technology* to help you sketch the graph of $y = 2x + \frac{100}{x}$.

c Find the minimum value of L and the corresponding value of x when this occurs.

d Sketch the optimum situation showing all dimensions.

3 A manufacturer can produce x fittings per day where $0 \leqslant x \leqslant 10\,000$. The production costs are:

- €1000 per day for the workers
- €2 per day per fitting
- €$\frac{5000}{x}$ per day for running costs and maintenance.

How many fittings should be produced daily to minimise the total production costs?

4 The total cost of producing x blankets per day is $\frac{1}{4}x^2 + 8x + 20$ dollars, and for this production level each blanket may be sold for $(23 - \frac{1}{2}x)$ dollars.
How many blankets should be produced per day to maximise the total profit?